Tim Hosgood (and Brendan Fong)
29th of August, 2025

TOPOS INSTITUTE

Abstractions for real people

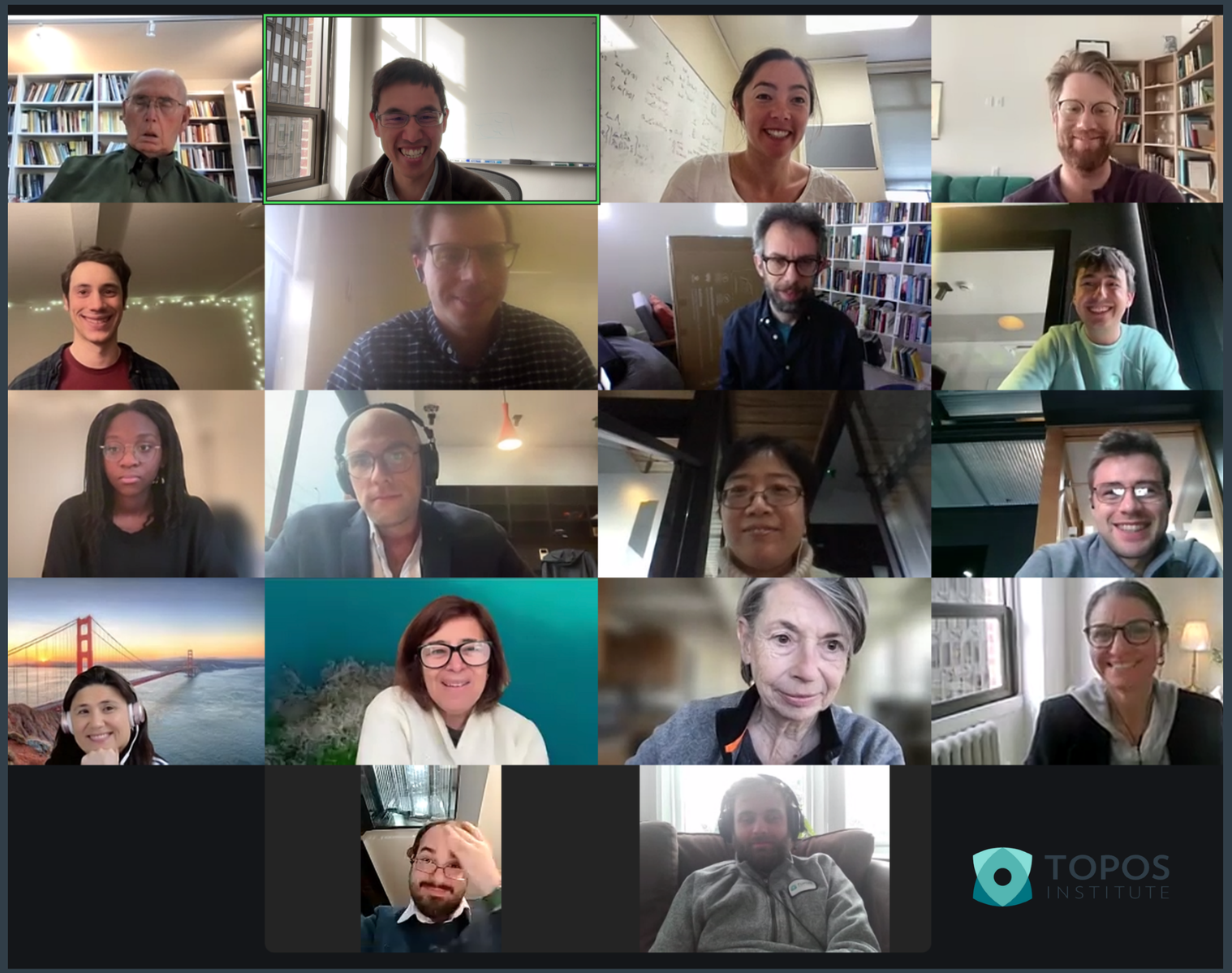
When do abstractions in maths and technology support people?

Claim

A practice of abstraction supports real people when it is **accessible**, **responsive**, and **pluralist**.

Smaller claim

Mathematicians can be helpful in realising such a practice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Holiday Party
Jan 2024

Three stories

[section one]

C Thi Nguyen: "The Limits of Data"

Issues in Science and Technology **XL(2)** (2024)

[...] every classification system represents some group's interests. Those interests are often revealed by where a classification system has fine resolution and where it doesn't. [The] International Classification of Disease (ICD) is a worldwide, standardized system for classifying diseases that's used in collecting mortality statistics, among other things. Without a centralized, standardized system, the data collected by various offices won't aggregate. But the ICD has highly variable granularity. It has separate categories for accidents involving falling from playground equipment, falling from a chair, falling from a wheelchair, falling from a bed, and falling from a commode. But it only has two categories for falls in the natural world: falling from a cliff, and an "other fall" category that lumps together all the other falls—including, in its example, falls from embankments, haystacks, and trees. The ICD is obviously much more interested in recording the kinds of accidents that might befall people in an urban industrial environment than a rural environment [...]

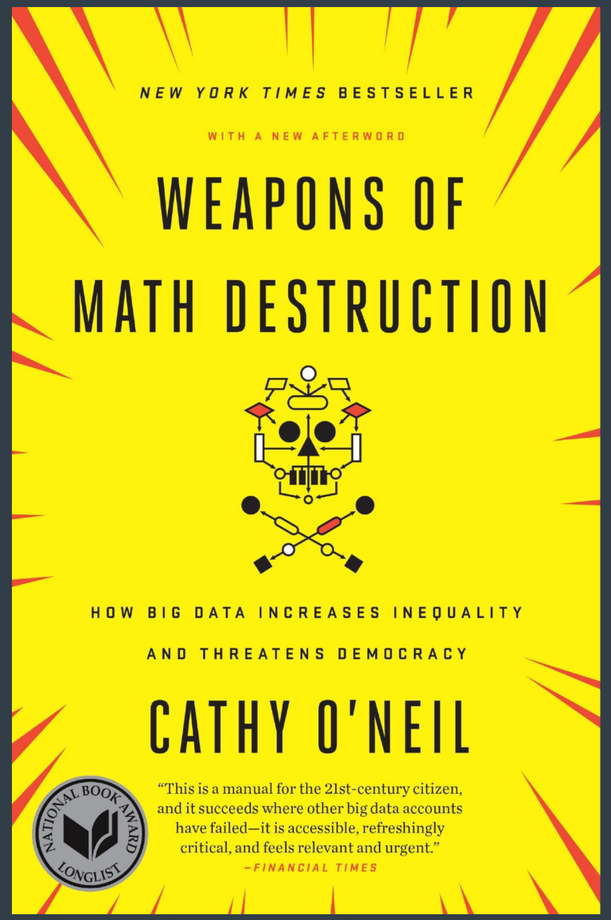
Cathy O'Neil: *Weapons of Math Destruction*

Crown Books, 2016

[The Chancellor of Washington DC Public Schools] developed a teacher assessment tool called IMPACT, and at the end of the 2009-10 school year the district fired all the teachers whose scores put them in the bottom 2 percent. […]

Sarah Wysocki, a fifth-grade teacher, didn't seem to have any reason to worry. […] One evaluation praised her attentiveness to the children; another called her "one of the best teachers I've ever come into contact with."

Yet at the end of the 2010-11 school year, Wysocki received a miserable score on her IMPACT evaluation. Her problem was a new scoring system known as value-added modeling, which purported to measure her effectiveness in teaching math and language skills. That score, generated by an algorithm, represented half of her overall evaluation, and it outweighed the positive reviews from school administrators and the community. This left the district with no choice but to fire her […]



Collin Lysford: "AI, The Idiot Ant Queen"

Desystemize, 2023

An abundance of human discretion dies twice: when we lose the ability to decide the world around us, and when we forget that the world was ever expected to be so responsive. On Christmas in 1914, spontaneous ceasefires sprouted along the trenches of World War 1. Back then, men had the freedom to choose not to kill. Do you think our new bomb computers have an exception for Christmas programmed in? Is spontaneous peace still possible? Or must peace be made legible to the whole military-algorithmic complex before it can be enacted? How many values in how many databases must be updated for killing to stop for a day? Is that number low or high? Is it getting larger each year? Do you know how you'd go about finding that number?

Do you think anyone knows it?

Abstraction

[section two]

What do I mean by *abstraction*?

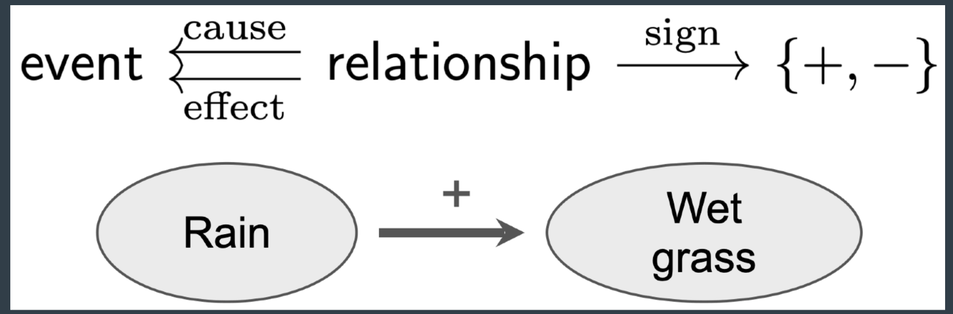
Distinction and judgements

- One way of describing the world **abstracts** another when it treats distinct expressions as the same.
- Abstraction is used to *focus attention* on distinctions relevant to the purpose at hand, and to make invisible irrelevant distinctions.
- Abstraction is an inherent act of *meaning making*, as it
  1. is done relative to a purpose, and
  2. makes judgements about what is relevant to that purpose.

What do I mean by *abstraction*?

Frames and contents

- A **frame** is a collection of abstractions assembled in such a way to provide language for a particular domain or worldview.
- Within a frame, **content** may be asserted as a specific description of a possible world within that frame.



Top: A simple **frame** for causality
Bottom: A specific **content** assertion

Abstraction in our three stories

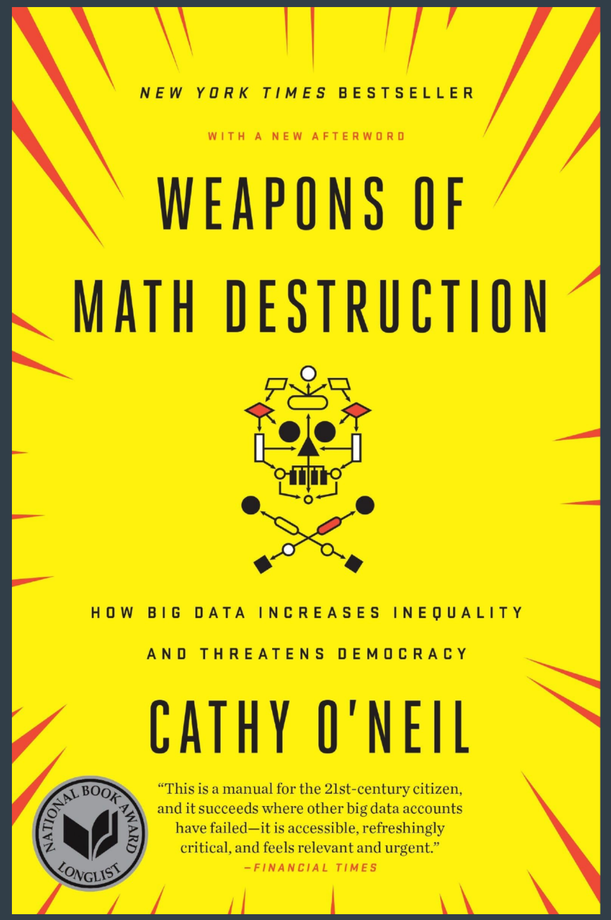
Nguyen, "The Limits of Data"

The problem occurs when users of data forget that categories are social inventions created for a purpose. When these classificatory schemes enter an information infrastructure, they become invisible; they become part of the background operating structure of our world. […] **The power of data is vast scalability; the price is context**.

Abstraction in our three stories

O'Neil, *Weapons of Math Destruction*

[There are] three elements of a Weapon of Math Destruction: **Opacity**, **Scale**, and **Damage**.



Abstraction in our three stories

Lysford, "AI, The Idiot Ant Queen"

[...] Institutions will always have gaps in their definitions that require squishy improvisation. The relevant variable here is whether a human is able to use their squishy improvisation skills to directly address the situation, or whether they need to use their squishy improvisation skills to try to change values in databases so the algorithmic expression of the institutions power hopefully, maybe, switches to doing the right thing. **This is about whether the interface between the institution and the world is a human or not.** This is about control.

What can mathematicians contribute?

Just one piece of the vast puzzle

Problem. The initial failure is that the underlying frame fails to capture something important to a community impacted by its use.

How can we help to remedy this?

What can *mathematicians* contribute?

Just one piece of the vast puzzle

Problem. The initial failure is that the underlying frame fails to capture something important to a community impacted by its use.

How can we help to remedy this?

Suggestion. Strive for a practice of abstraction in which people can

1. **Name the frame**, articulating when and why content in a specific frame cannot represent them
2. **Reshape the frame** according to their needs
3. Work effectively with others who **prefer different frames**.

What can *mathematicians* contribute?

Just one piece of the vast puzzle

Problem. The initial failure is that the underlying frame fails to capture something important to a community impacted by its use.

How can we help to remedy this?

Suggestion. Strive for a practice of abstraction which is

1. **accessible**
2. **responsive**
3. **pluralist**.

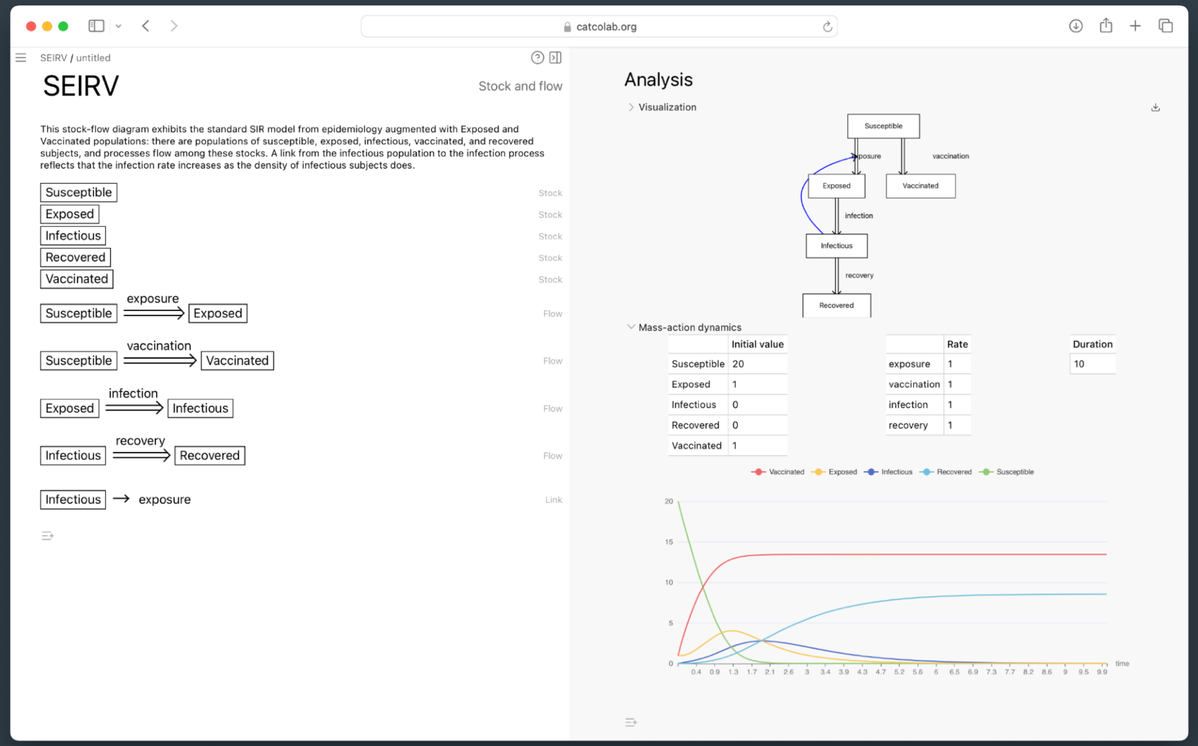
CatColab

[section three]

CatColab

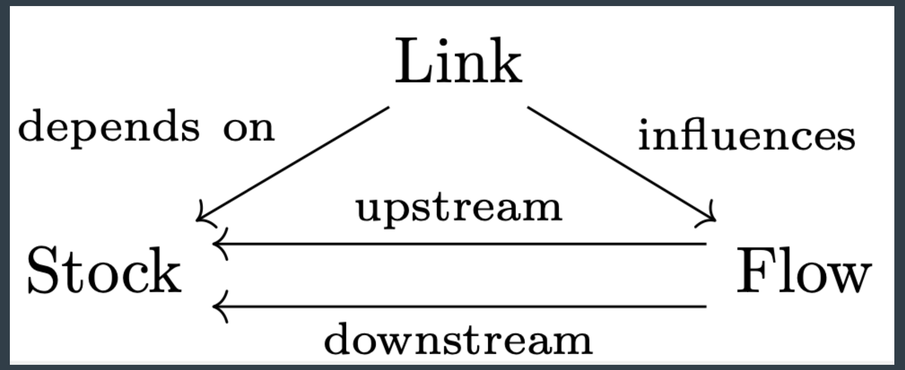
Software and mathematics from the Topos Institute

- CatColab is a platform for collective sense-making via formal, interoperable, conceptual modelling
- Some aspects of Jupyter, SQL, Google Sheets, Lean, GitHub, …
- Open-source open-access
- Research software, but built for (*and with*) specific users: researchers, modellers, policy-makers; public health, food safety, biopharmacology, …



Stock-flow diagrams

An example from system dynamics



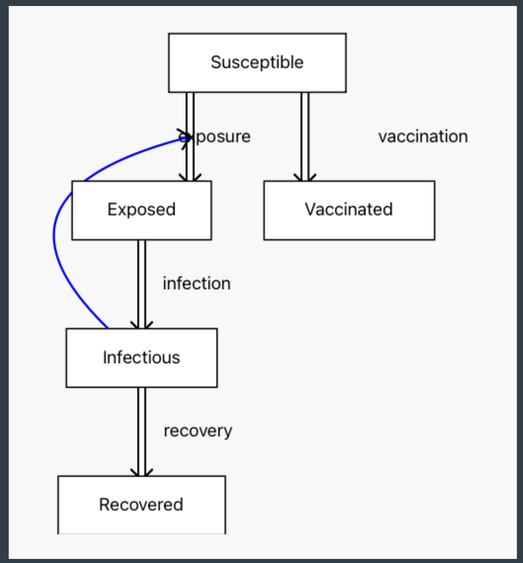
Frame

SEIRV

Stock and flow

This stock-flow diagram exhibits the standard SIR model from epidemiology augmented with Exposed and Vaccinated populations: there are populations of susceptible, exposed, infectious, vaccinated, and recovered subjects, and processes flow among these stocks. A link from the infectious population to the infection process reflects that the infection rate increases as the density of infectious subjects does.

Susceptible	Stock
Exposed	Stock
Infectious	Stock
Recovered	Stock



Content

Causal loop diagrams

Another example from system dynamics

rabbits and foxes 💭

An exercise in participatory modelling

CatColab as frames and content

Double categories and laxity

- The underlying framework is a declarative language based on double-categorical logic.
- Every machine-readable statement has a formal mathematical semantics.

CatColab as frames and content

Double categories and laxity

Layer	Mathematical semantics	Examples
Doctrine	types of ambient structured double category	double categories, double categories with tabulators, cartesian double categories
Theory	(structured) double category $\mathbb{D}$	causal loop diagrams, database schema, stock-flow models, Petri nets, discrete exterior calculus
Model	(structured) lax double functor $M : \mathbb{D} \to \mathbb{S}\mathrm{pan}$	database schema for the ICD, stock-flow SEIRV model, regulatory network for drug degredation
Instance	(structured) bimodule $I : 1 \to M$	database of health outcomes in Nairobi City County according to the ICD database schema

Accessible, responsive, pluralist

[section four]

1. Accessible

Suggested definition

A practice of abstraction is **accessible** to a (person or) community if it is clear what each frame is tracking, and it is clear what it is not.

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Suggested definition

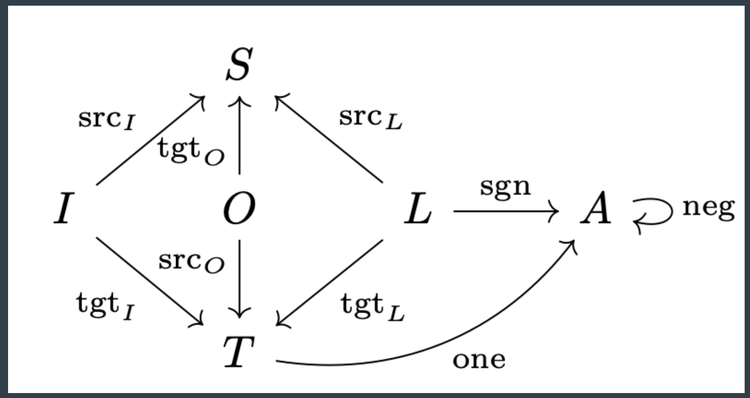
A practice of abstraction is **accessible** to a (person or) community if it is clear what each frame is tracking, and it is clear what it is not.

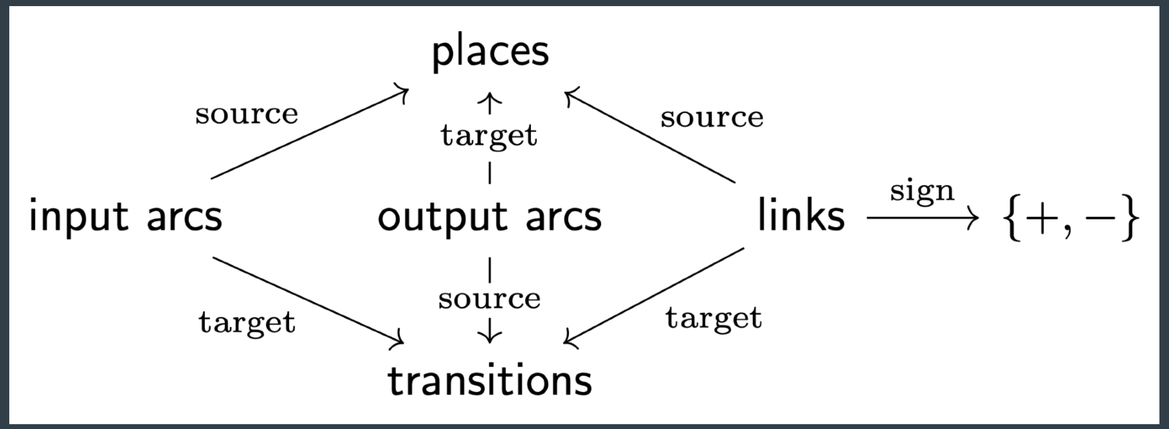
- Note that transparency is *relative to a community*.
- *Mathematicians* use formality to increase accessibility; as frames get more complex, we need to be thoughtful about how this works.

1. Accessible

Formality as a tool for accessibility *to mathematicians*

"A **Petri net** consists of **places**, **transitions**, and **arcs**. Arcs run from a place to a transition or vice versa, never between places or between transitions. The places from which an arc runs to a transition are called the **input places** of the transition; the places to which arcs run from a transition are called the **output places** of the transition. […]"



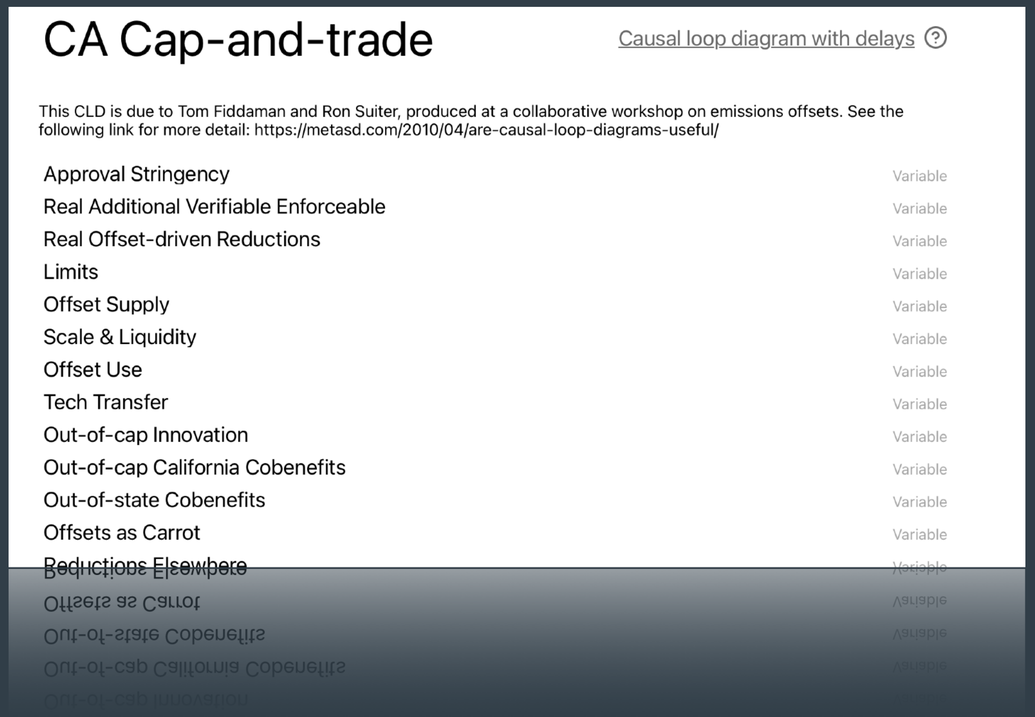


1. Accessible

Formality for enabling computational tools for accessibility

Formality can provide a basis for accessibility via the *computational embodiment* of the abstraction.

Accessibility may be increased through visualisation, automated reasoning, simulation, ...



2. Responsive

Suggested definition

A practice of abstraction is **responsive** if a frame can be modified without undue cost.

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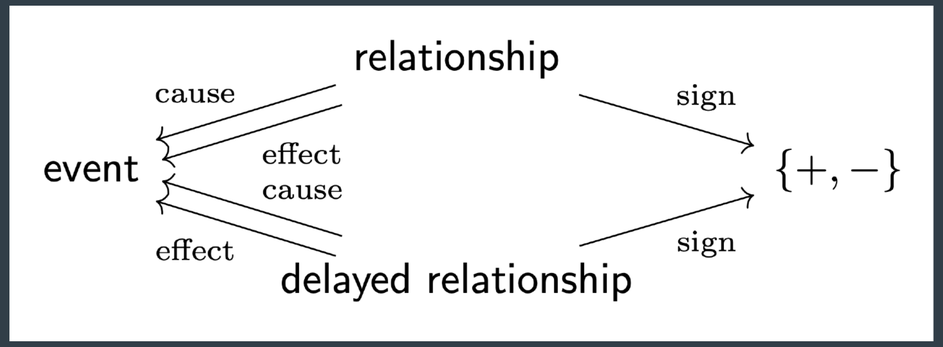
A practice of abstraction is **responsive** if a frame can be modified without undue cost.

- Note this is *relative to the level of resources* (funding, time, expertise,…) *of a community.*
- One driver of cost is loss of content: to change frame, one often must reconstruct content from scratch.
  - It’s nontrivial to port a climate model from Fortran to Julia
  - It’s difficult to connect with the field of macroeconomics by varying the dynamic stochastic general equilibrium paradigm

2. Responsive

Explicit frames as a tool for responsiveness

Frame for causality

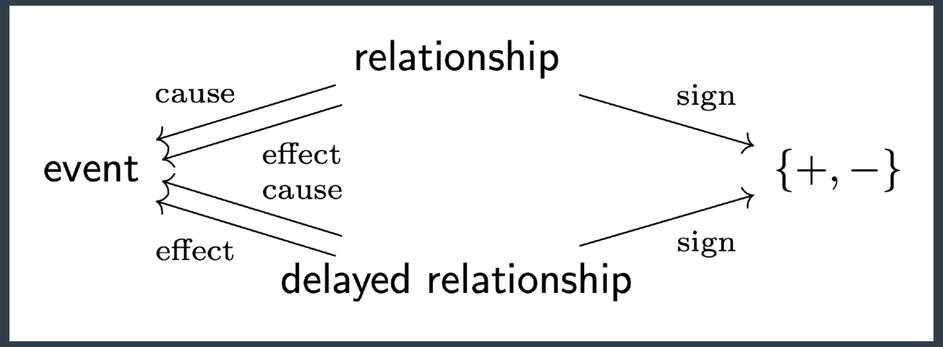


Frame for causality with delays

2. Responsive

Explicit frames as a tool for responsiveness

Frame for causality



Frame for causality with delays

strawberries 💭

3. Pluralist

Suggested definition

A practice of abstraction is **pluralist** if multiple distinct frames can be used simultaneously in a coherent and productive way.

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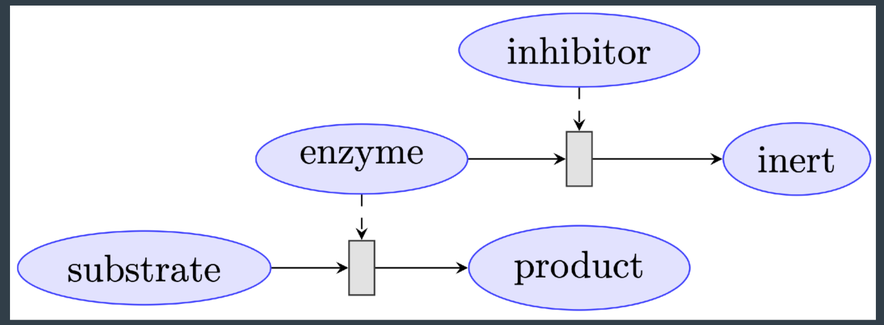
Suggested definition

A practice of abstraction is **pluralist** if multiple distinct frames can be used simultaneously in a coherent and productive way.

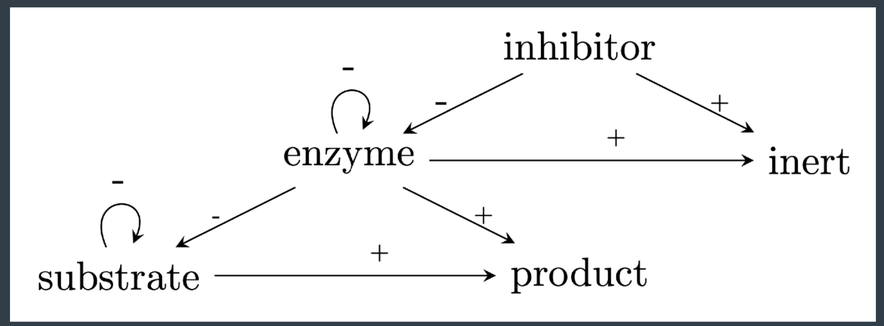
- A good frame is about directing attention to what matters and away from what doesn't: a choice of good frame is relative to *what a community cares about*, and its *attentional bandwidth*.
  - In particular, there is a tradeoff between expressivity (more potential distinctions) and reasoning/support (fewer irrelevant distinctions).
- Without an ability to work with pluralism, communities come into conflict about which frame to use — the search for "one true frame".

3. Pluralism

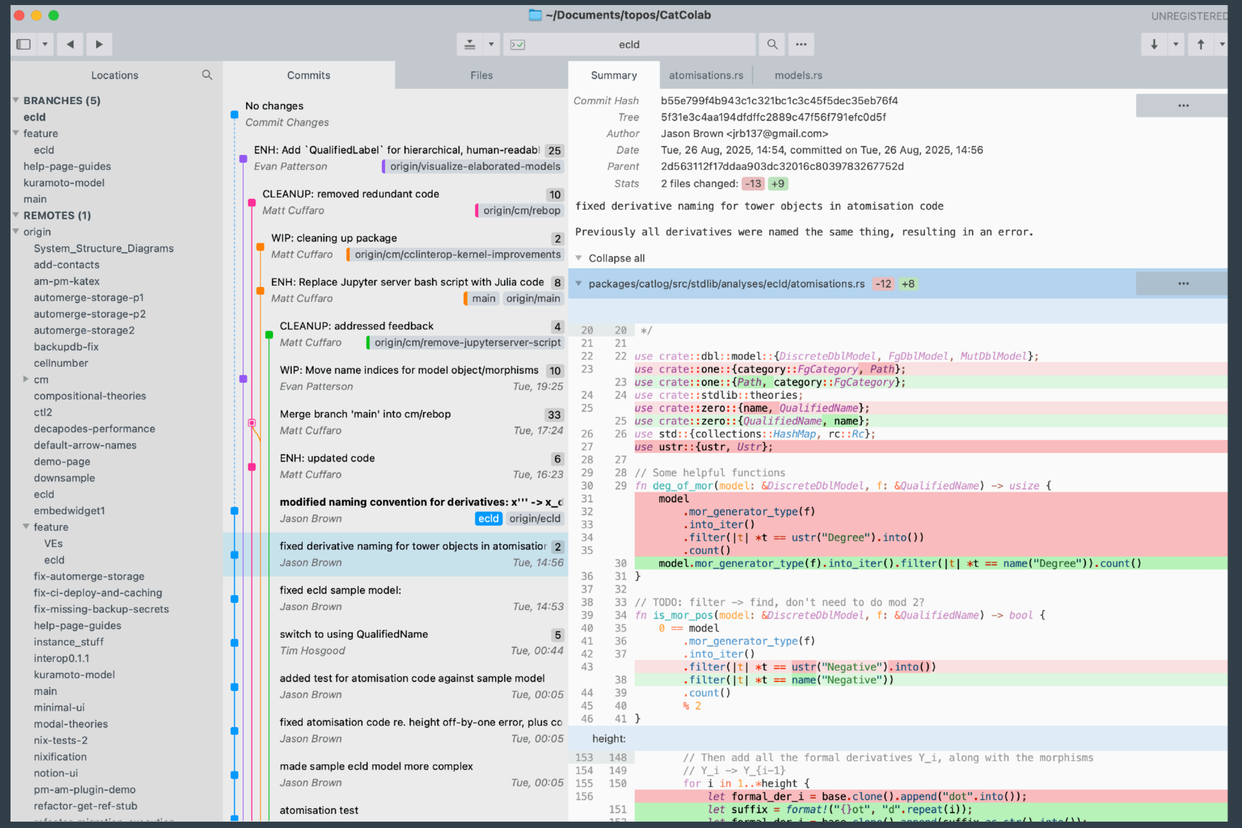
Migrations and version control



Enzyme inhibition as a Petri net with links



Enzyme inhibition as a causal loop diagram



Version control

Claim

A practice of abstraction supports real people when it is **accessible**, **responsive**, and **pluralist**.

Smaller claim

Mathematicians can be helpful in realising such a practice